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Abstract

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SIGNIFICANCE TEST FOR A PARTIAL CORRELATION

CORRECTED FOR ATTENUATION*

When each of two variables is contaminated by a third variable, it often is important in studying the relationship between the first two to "hold constant" or partial out the third. For example, scores on each of two personality tests may be unavoidably contaminated by general intelligence, by "attitude acquiescence," or by some other cognitive ability or personality trait. If we want the correlation between the first two personality traits, contaminating variables must be partialed out. For numerous examples, see Stricker, Messick, and Jackson (1968).

The population partial $\rho_{xy\cdot z}$ or sample partial $r_{xy\cdot z}$ between x and y with z "held constant" is by definition the correlation of the residuals in the linear prediction of x from z with the residuals in the linear prediction of y from z. Since partial correlations are regarded with distrust by many behavioral scientists, it is worth pointing out that <u>if the regression of x and y on z is linear and if the conditional distribution of the residuals is independent of z, then the partial $\rho_{xy\cdot z}$ is numerically equal to the simple zero-order correlation between x and y computed for all observations having an arbitrarily chosen value z; this zero-order correlation is the same no matter what arbitrary value of z is chosen</u>. Thus, in this case, use of the partial correlation really eliminates the concomitants of variation in z. If the residuals are not distributed independently of z, the partial is a

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sort of weighted average of the correlations between x and y for fixed x , averaged over the different fixed values of z .

An important problem arises because the presence of errors of measurement in z can cause the partial correlation $\rho_{xy \cdot z}$ to be negative when otherwise it would be positive. If the errors of measurement are uncorrelated with each other and with all other variables, their effects can be removed by correcting for attenuation each zero-order correlation involving z . This results, after a little algebra, in a formula given and discussed by Stouffer (1936, eq. 9), Saunders (1951), Lord (1963, eq. 23), Kahneman (1965), Bohrnstedt (1969, eq. 21), Bergman (1971), and others:

$$\rho_{xy \cdot \zeta} = \frac{\rho_{xy}\rho_{zz'} - \rho_{xz}\rho_{yz}}{\sqrt{\rho_{zz'} - \rho_{xz}^2}\,\sqrt{\rho_{zz'} - \rho_{yz}^2}} , \tag{1}$$

where each ρ is a population correlation, ζ represents z without errors of measurement, and a prime denotes a score on a parallel form of a test. It is assumed that $\rho_{zz'} > 0$.

If it is desired to correct also for the effects of errors of measurement in x and y , the corrected partial is

$$\rho_{\xi\eta \cdot \zeta} = \frac{\rho_{xy}\rho_{zz'} - \rho_{xz}\rho_{yz}}{\sqrt{\rho_{xx'}\rho_{zz'} - \rho_{xz}^2}\,\sqrt{\rho_{yy'}\rho_{zz'} - \rho_{yz}^2}} , \tag{2}$$

where ξ and η represent x and y without errors of measurement. The correction for errors in x and y can only increase the magnitude of the partial; it cannot cause a change in sign.

Good small-sample procedures for setting up confidence intervals for $\rho_{xy\cdot\zeta}$ or for $\rho_{\xi\eta\cdot\zeta}$ are not currently available. The present paper considers the most obvious large-sample procedure and suggests an often useful alternative.

Sufficient Statistics

Since z and z' are parallel, we know that certain variances and covariances are equal: $\sigma_z^2 = \sigma_{z'}^2$, $\sigma_{xz} = \sigma_{xz'}$, $\sigma_{yz} = \sigma_{yz'}$. The problem of how best to estimate $\rho_{xy\cdot\zeta}$ or $\rho_{\xi\eta\cdot\zeta}$ is clarified if we consider the new variables $u = z + z'$ and $v = z - z'$.

The variance-covariance matrix of the variables x , y , u , v is

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} & 2\sigma_{xz} & 0 \\ \sigma_{xy} & \sigma_y^2 & 2\sigma_{yz} & 0 \\ 2\sigma_{xz} & 2\sigma_{yz} & 2(\sigma_z^2 + \sigma_{zz'}) & 0 \\ 0 & 0 & 0 & 2(\sigma_z^2 - \sigma_{zz'}) \end{bmatrix} \tag{3}$$

If x , y , z , and z' have a multivariate normal distribution, as will be assumed hereafter, then by (3) v is independent of x , y , and u . The seven parameters σ_x^2 , σ_y^2 , σ_u^2 , σ_v^2 , σ_{xy} , σ_{xu} , σ_{yu} can best be estimated from the usual sample variances and covariances s_x^2 , s_y^2 , s_u^2 , s_v^2 , s_{xy} , s_{xu} , s_{yu} ; for these last can now be seen [Anderson, 1958, section 3.3.3] to be sufficient statistics for this purpose.

The formulas in (3) can be used to write parameters involving z in terms of those involving u and v :

$$\left.\begin{aligned}
\sigma_z^2 &= (\sigma_u^2 + \sigma_v^2) / 4 \\
\sigma_{zz'} &= (\sigma_u^2 - \sigma_v^2) / 4 \\
\sigma_{xz} &= \sigma_{xu} / 2 \\
\sigma_{yz} &= \sigma_{yu} / 2
\end{aligned}\right\} \qquad (4)$$

Thus, s_x^2 , s_y^2 , s_u^2 , s_v^2 , s_{xy} , s_{xu} , and s_{yu} are sufficient statistics for the parameters in (1) and (2) . Any estimation procedure or hypothesis test concerning (1) or (2) should depend on these seven sufficient statistics alone. The sufficient statistics are easily calculated with the help of the formulas

$$\left.\begin{aligned}
s_u^2 &= s_z^2 + s_{z'}^2 + 2s_{zz'} \\
s_{xu} &= s_{xz} + s_{xz'} \\
s_{yu} &= s_{yz} + s_{yz'} \\
s_v^2 &= s_z^2 + s_{z'}^2 - 2s_{zz'}
\end{aligned}\right\} \qquad (5)$$

Estimation

Using (4) , the four parameters in (1) are $\rho_{xy} = \sigma_{xy} / \sigma_x\sigma_y$ and

$$\left.\begin{aligned}
\rho_{xz} &= \frac{\sigma_{xz}}{\sigma_x\sigma_z} = \frac{\sigma_{xu}}{\sigma_x\sqrt{(\sigma_u^2 + \sigma_v^2)}} \\
\rho_{yz} &= \frac{\sigma_{yz}}{\sigma_y\sigma_z} = \frac{\sigma_{yu}}{\sigma_y\sqrt{(\sigma_u^2 + \sigma_v^2)}} \\
\rho_{zz'} &= \frac{\sigma_{zz'}}{\sigma_z^2} = \frac{\sigma_u^2 - \sigma_v^2}{\sigma_u^2 + \sigma_v^2}
\end{aligned}\right\} \qquad (6)$$

Substituting s for σ in (6) and using (5) produces the estimators

$$\left.\begin{aligned}
\hat{\rho}_{xy} &= s_{xy} / s_x s_y \text{ and} \\
\hat{\rho}_{xz} &= \frac{s_{xz} + s_{xz'}}{s_x\sqrt{(2s_z^2 + 2s_{z'}^2)}} \\
\hat{\rho}_{yz} &= \frac{s_{yz} + s_{yz'}}{s_y\sqrt{(2s_z^2 + 2s_{z'}^2)}} \\
\hat{\rho}_{zz'} &= \frac{2s_{zz'}}{s_z^2 + s_{z'}^2}
\end{aligned}\right\} \qquad (7)$$

These estimators are not unbiased; however, the corresponding unbiased estimators would be excessively complicated.

If we substitute the $\hat{\rho}$ of (7) for the ρ on the right side of (1), the result after some simplification can be written

$$\hat{\rho}_{xy.\zeta} = \frac{4s_{xy}s_{zz'} - s_{xu}s_{yu}}{\sqrt{(4s_x^2 s_{zz'} - s_{xu}^2)}\sqrt{(4s_y^2 s_{zz'} - s_{yu}^2)}} \quad . \qquad (8)$$

Expressed in terms of sample correlation coefficients (r) instead of covariances, this becomes

$$\hat{\rho}_{xy.\zeta} = \frac{r_{xy}r_{uu'} - r_{xu}r_{yu}}{\sqrt{(r_{uu'} - r_{xu}^2)}\sqrt{(r_{uu'} - r_{yu}^2)}} \quad , \qquad (9)$$

where by definition

$$r_{uu'} = \frac{4s_{zz'}}{s_z^2 + s_{z'}^2 + 2s_{zz'}} \quad . \qquad (10)$$

Equations (8) and (9) seem to be good practical formulas to use for estimating $\hat{\rho}_{xy\cdot\zeta}$ when the data include parallel tests z and z' .

Formula (10) is one of several formulas used to estimate the reliability of a double-length test (Cronbach, 1951). It has been attributed to Flanagan (Kelley, 1942). It is interesting to note that the rather lengthy reasoning and algebra given here leads to an estimator of $\rho_{xy\cdot\zeta}$ that is identical with (1) except that ρ is replaced by r <u>and that z is replaced by the double-length test $u = z + z'$</u> .

<u>Sampling Fluctuations in $\hat{\rho}_{xy\cdot\zeta}$</u>

The statistics s_x^2 , s_y^2 , s_u^2 , s_v^2 , s_{xy} , s_{xu} , s_{yu} are the same as the maximum likelihood estimators of the corresponding parameters. It follows that (8) or (9) is an asymptotically efficient estimator for $\hat{\rho}_{xy\cdot\zeta}$.

The assumptions made about errors of measurement guarantee that the denominator of $\rho_{xy\cdot\zeta}$ in (1) will not be imaginary (Lord & Novick, 1968, eq. 3.9.8). Unfortunately, there is no such guarantee for $\hat{\rho}_{xy\cdot\zeta}$, the estimated partial correlation in (8) or (9). Thus, for example, sampling fluctuations in $r_{uu'}$, r_{xu} , or r_{yu} could cause $\hat{\rho}_{xy\cdot\zeta}$ defined by (9) to be infinite. In practice, $\hat{\rho}_{xy\cdot\zeta}$ defined by (9) may show very large sampling fluctuations if the sample is too small, especially if either x or y is highly correlated with the true score ζ .

The sampling fluctuations of (8) or (9) will in some (not all) cases be so large as to make the calculation of $\hat{\rho}_{xy\cdot\zeta}$ almost useless. For any small sample, a Monte Carlo study might be required to evaluate the seriousness of the sampling fluctuations. Bergman (1971) carried

out a Monte Carlo study and found that for his data, the sample partials corrected for attenuation were less biased than the uncorrected sample partials for estimating the corrected population partials; but the uncorrected sample partials were closer to the population corrected partials than the corrected sample partials in the sense of having lower mean squared error.

Statistical Inference about the Sign of the Corrected Population Partial Correlation

Even if we cannot estimate the size of the corrected population partial correlation, we may still be able to assert that it is positive, or that it is negative. The numerator of (1) has the same sign as $\rho_{xy\cdot\zeta}$ and $\rho_{\xi\eta\cdot\zeta}$. Thus the hypotheses that $\rho_{xy\cdot\zeta}$ and $\rho_{\xi\eta\cdot\zeta}$ are greater than, equal to, or less than zero are indistinguishable from the same hypotheses about the numerator $\rho_{xy}\rho_{zz'} - \rho_{xz}\rho_{yz}$. If our main concern is to infer the sign of the corrected population partial, we can restrict our attention to one-sided and two-sided tests of the null hypothesis

$$H_0 : \rho_{xy}\rho_{zz'} = \rho_{xz}\rho_{yz} \tag{11}$$

against the alternative hypotheses $\rho_{xy}\rho_{zz'} > \rho_{xz}\rho_{yz}$ or $\rho_{xy}\rho_{zz'} < \rho_{xz}\rho_{yz}$.

Since z and z' are parallel, the null hypothesis can be rewritten either

$$\left.\begin{array}{l} H_0 : \rho_{xy}\rho_{zz'} - \rho_{xz}\rho_{yz'} = 0 \\ \text{or} \\ H_0 : \rho_{xy}\rho_{zz'} - \rho_{xz'}\rho_{yz} = 0 \end{array}\right\} \tag{12}$$

(as well as other ways).

The left side of each equation in (12) is a tetrad difference (Spearman, 1927). It might seem natural to base a significance test on the sample tetrads $r_{xy}r_{zz'} - r_{xz}r_{yz'}$ and $r_{xy}r_{zz'} - r_{xz'}r_{yz}$. Kelley (1928, 1947, eq. 11.42) gave the asymptotic sampling variance of a tetrad. When H_0 holds and z and z' are parallel, his formula becomes

$$\begin{aligned}\mathrm{Var}(r_{xy}r_{zz'} - r_{xz}r_{yz'}) = \frac{1}{N-2}(&\rho_{xy}^2 + \rho_{xz}^2 + \rho_{yz}^2 + \rho_{zz'}^2 \\ &+ 2\rho_{xy}\rho_{xz}\rho_{yz}\rho_{zz'} + 2\rho_{xz}^2\rho_{yz}^2 - 4\rho_{xy}\rho_{xz}\rho_{yz} \\ &- 2\rho_{xz}^2\rho_{zz'} - 2\rho_{yz}^2\rho_{zz'}) \quad .\end{aligned} \tag{13}$$

Better yet, Hotelling (1936) found the exact distribution of a sample tetrad difference. Unfortunately, even this does not solve the problem, since we have available two equally relevant sample tetrad differences. The information from both must somehow be taken into account, but they are certainly not independently distributed.

Lacking any suitable exact sampling distribution, it seems necessary to resort to some asymptotic approximation. A likelihood ratio significance test could be derived, but this would require iterative solution of the likelihood equations. A more convenient significance test is desired here.

Asymptotic Significance Test

If (12) is multiplied by $\sigma_x\sigma_y\sigma_z^2$, H_0 can be written

$$H_0 : \sigma_{xy}\sigma_{zz'} - \sigma_{xz}\sigma_{yz} = 0 \quad . \tag{14}$$

It follows from a general theorem (see Moran, 1970) that an asymptotically optimal significance test of H_0 results when

1. each σ in (14) is replaced by its maximum likelihood estimate,

2. the asymptotic standard error of the resulting statistic is approximated by substituting maximum likelihood estimates for the parameters in the appropriate formula,

3. the statistic from step 1 is divided by the approximate standard error from step 2,

4. the quotient is treated as a normally distributed variable with zero mean and unit variance.

For the foregoing, the maximum likelihood estimates are obtained without the restraint imposed by the null hypothesis.

Step 1 is carried out by substituting (4) in (14), replacing σ by s, and using (5). Denoting the resulting statistic by T, we have

$$T = s_{xy}s_{zz'} - \frac{1}{4}(s_{xz} + s_{xz'})(s_{yz} + s_{yz'}) \quad . \tag{15}$$

Since by (10) $s_{zz'} = s_u^2 r_{uu'}/4$, T may be rewritten

$$T = \frac{1}{4} s_x s_y s_u^2 (r_{xy}r_{uu'} - r_{xu}r_{yu}) \quad . \tag{16}$$

The formula for the approximation to the standard error of T is found by the delta method (Kendall & Stuart, 1958, section 10.6) to be

$$\begin{aligned} S.E._T = \frac{1}{\sqrt{(2N)}} \sigma_x\sigma_y\sigma_z^2 (2\rho_{xy}^2 + \rho_{xz}^2 + \rho_{yz}^2 + 2\rho_{zz'}^2 \\ - 6\rho_{xy}\rho_{xz}\rho_{yz} - 3\rho_{xz}^2\rho_{zz'} - 3\rho_{yz}^2\rho_{zz'} \\ + 4\rho_{zz'}^2\rho_{xy}^2 + 8\rho_{xz}^2\rho_{yz}^2 - 6\rho_{xz}\rho_{yz}\rho_{xy}\rho_{zz'})^{1/2} \quad , \end{aligned} \tag{17}$$

where N is the number of observations. We can rewrite the correlations in (17) in terms of variances and covariances, then substitute (4) into (17), replace σ by s throughout, and substitute from (5), obtaining the approximate standard error formula

$$\begin{aligned} S.E._T \doteq \frac{1}{4\sqrt{N}} s_x s_y s_u^2 [4r_{xy}^2 + r_{xu}^2 + r_{yu}^2 + r_{uu'}^2 \\ - 6r_{xy}r_{xu}r_{yu} - 2r_{uu'}r_{xu}^2 - 2r_{uu'}r_{yu}^2 \\ - 4r_{uu'}r_{xy}^2 + 3r_{xy}^2 r_{uu'}^2 + 4r_{xu}^2 r_{yu}^2]^{1/2} \quad . \end{aligned} \tag{18}$$

From (16) and (18), the asymptotic optimal test statistic is, finally

$$\frac{T}{S.E._T} \doteq \frac{(r_{xy}r_{uu'} - r_{xu}r_{yu})\sqrt{N}}{\begin{aligned}(4r_{xy}^2 + r_{xu}^2 + r_{yu}^2 + r_{uu'}^2 - 6r_{xy}r_{xu}r_{yu} - 2r_{uu'}r_{xu}^2 \\ - 2r_{uu'}r_{yu}^2 - 4r_{uu'}r_{xy}^2 + 3r_{xy}^2 r_{uu'}^2 + 4r_{xu}^2 r_{yu}^2)^{1/2}\end{aligned}} \quad . \tag{19}$$

The values of r_{xu}, r_{yu}, and $r_{uu'}$ are to be calculated from (10) and (5).

The asymptotic properties of (19) may not be much better than the asymptotic properties of $\hat{\rho}_{xy\cdot\zeta}$/(estimated asymptotic standard error of $\hat{\rho}_{xy\cdot\zeta}$), but there is one important advantage for (19). Since for finite N the denominator of $\hat{\rho}_{xy\cdot\zeta}$ can be zero, the true sampling variance of $\hat{\rho}_{xy\cdot\zeta}$ is necessarily infinite. No such problem arises with the statistic T of (16). The superiority of (19) should be apparent in a Monte Carlo study, such as Bergman (1971) carried out for $\hat{\rho}_{xy\cdot\zeta}$.

Numerical Example

Suppose $r_{xy} = .30$, $r_{xz} = r_{xz'} = r_{yz} = r_{yz'} = .20$, $r_{zz'} = .60$, $s_z = s_{z'}$ and $N = 100$. We find that $r_{uu'} = .75$ and $r_{xu} = r_{yu} = \sqrt{(.05)}$. Also by (9) that

$$\hat{\rho}_{xy\cdot\zeta} = \frac{.225 - .05}{.75 - .05} = .25 \quad .$$

By (19),

$$\frac{T}{S.E._T} \doteq \frac{(.225 - .05)\sqrt{(100)}}{\sqrt{(.36 + .05 + .05 + .5625 - .09 - .075 - .075 - .27 + .151875 + .01)}}$$

$$\doteq 2.13 \quad .$$

Since 5 percent of the normal curve frequency lies above a relative deviate of 1.64 and since 2.5 percent lies above 1.96, we reject the null hypothesis at the .05 level regardless of whether a one-tailed or a two-tailed significance test is used. We conclude that the true corrected partial is positive. If we must estimate its numerical value, we estimate $\rho_{xy\cdot\zeta} = .25$.

Summary

Since a partial correlation among true scores may have the opposite algebraic sign from the corresponding partial correlation among observed scores, it can be important to correct for attenuation. Formulas (8), (9) based on sufficient statistics are given for estimating the corrected partial correlation.

Under certain circumstances, the estimated corrected partial correlation may be subject to overwhelming sampling errors. For the situation where the main requirement is to infer the algebraic sign of the corrected partial correlation rather than its numerical value, a significance test that avoids this difficulty is presented (19).

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